



## ECBA-17

### Optimization of Central Patterns Generators

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#### Abstract

The issue of how best to optimize Central Patterns Generators (CPG) for locomotion to generate motion for one leg with two degrees of freedom has inspired many researchers to explore the ways in which rhythmic patterns obtained by genetic algorithms may be utilized in uncoupled, unidirectional and bidirectional two CPGs. This paper takes as its assumption that the focus on stability analysis to decrease variation between steps brings about better results with respect to the gait locomotion, and argues that controlling the amplitude and frequency may lead to more robust results viz., stimulation for movement.

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*Keywords*—Central Patterns Generators (CPGs), Kinematic Model of One Leg, Stability, Optimizing Gait Generation

#### Introduction

Recent studies on stimulation for movement, such as walking, swimming and running have shown that the basic locomotor patterns of biological systems are produced by a central nervous system, referred to as the Central Pattern Generator (CPG) (Larsen; Sillar, 1996). Central Pattern Generators are biologically inspired networks of nonlinear oscillating neurons that are capable of producing rhythmic patterns without sensory feedback. Localized in the spinal cord of animals, the CPG sends signals from the brainstem to produce a periodic activity, and hence generates rhythmic commands for the muscles (Brown, 1911); (Ijspeert, 2008); (Ijspeert, Crespi, Ryczko, & Cabelguen, 2007); (Sproewitz, Moeckel, Maye, & Ijspeert, 2008). Recent studies on human body have shown that many functions that cannot be controlled by the human body consciously are controlled by the CPGs, such as breathing and digestion (Billard & Ijspeert, 2000).

Generally speaking, CPGs are considered a set of nonlinear oscillators and each of the set of nonlinear oscillators is forced by the output of a sensor, which gives a time-index to the first-order information on the motion (Ijspeert, 2008). A neural oscillator is formed by two neurons with inhibitive connections between them and the responses of two neurons of a neural oscillator suppress each other in such a way that one of them is extensor neuron and the other is flexor neuron (Bucher, Haspel, Golowasch, & Nadim, 2000); (Casasnovas & Meyrand, 1995); (Van Vreeswijk, Abbott, & Ermentrout, 1994).

Interestingly, many physical structures of the limbs and arms have been modeled, and the control systems have been copied to regenerate the same move patterns in the robots as seen in nature. CPGs always synchronize with body movement and accordingly burst rhythmic patterns to motor neurons at an appropriate time in a movement cycle (Ijspeert, 2008). In legged locomotion, each leg is controlled by distinct neuronal network, where the CPG gives signals to each joint (Amrollah & Henaff, 2010; Ijspeert, 2008). Experiments reveal that there is a tight coupling between sensory feedback and CPGs. The reflexes are phase-dependent, they will have different effects depending on the timing within locomotor cycle (Pearson, 1995). Various models of CPG used for controlling the biped locomotion in human robots have been introduced (for details, see, Aoi & Tsuchiya, 2005; Endo et al., 2005; (Taga, 1998); (Taga, Yamaguchi, & Shimizu, 1991). Different modes of locomotion have been controlled by Models of CPGs, such as the CPG models used with octopod and hexapod robots inspired by insect locomotion (Arena et al., 2004; Inagaki et al., 2006; Inagaki, Yuasa, & Ari, 2003). CPGs have been also used to control swimming robots, such as swimming lamprey or eel robots (see, Arena, 2001; Crespi, & Ijspeert, 2008; Ijspeert, & Crespi, 2007; Inagaki et al., 2006), as well as to control Quadruped robots (for details, see, Billard & Ijspeert, 2000; Brambilla, Buchli, & Ijspeert, 2006; Fukuoka, Hikmura, & Cohen, 2003). This paper summarizes the kinematics model used for simulations and gaits design, explains the Uncouple, Unidirectional and Bidirectional two CPGs structures, and analyze stability of the

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mode. It also explores how optimized Central Pattern Generator structures may be adapted to robotic systems that perform one-leg movement, and gives suggestions for future research.

### Kinematic Model

Kinematic model is designed to perform basic analysis. Figure1 shows the flight and stance modes of the leg structure, where  $L_1, L_2$  represent the lengths of the thigh and the calf leg respectively, and  $\theta_1, \theta_2$  show the angular positions of the hip and the knee. Let us also assume that  $(x_A, y_A)$  denotes the first coordinate of the hip, and  $(x_f, y_f)$  denotes the second coordinate of the knee. Now, if the tip of the second link touches the ground, the leg will behave like a revolute joint. This indicates that a zero slip is considered between the tip of the link and ground surface. As such, the body will move along x-direction only in stance mode.

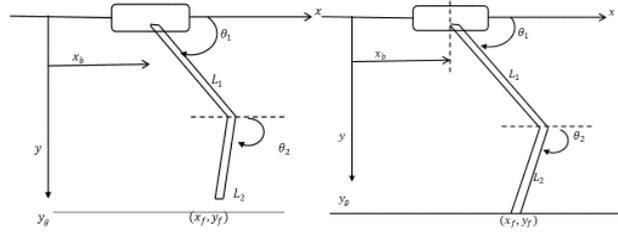


Figure 1: Leg System in Swing and Stance Mode

As Figure 1 shows, we have two cases: The first case is when the leg is in stance mode, the Kinematic Model has one degree of freedom. The hip joint angle  $\theta_1$  is also calculated with respect to knee angle  $\theta_2$ , which is determined by the CPG. The second case is when the leg is in swing mode, we obtain leg with 2 DOF. The hip and knee joint angles are calculated by Uncoupled, Unidirectional and Bidirectional two CPGs. The kinematic equations are

$$\begin{aligned} x_A &= x_b + L_1 \cos \theta_1, & x_f &= x_b + L_1 \cos \theta_1 + L_2 \cos \theta_2, \\ y_A &= L_1 \sin \theta_1 & y_f &= L_1 \sin \theta_1 + L_2 \sin \theta_2 \end{aligned}$$

### Central Pattern Generators (CPGs)

As defined previously, CPGs are biologically inspired networks of nonlinear oscillating neurons that are cable of producing rhythmic patterns without sensory feedback. Recently, a plethora of applications have been implemented using different neutrals in robotic structures. These neutrals are implemented by software methods called CPGs, where the CPG unit is responsible for generating required angular references for the hip and knee joints. The mathematical differential equations present the CPGs in general formula ((Larsen)(Ijspeert & Crespi, 2007); (Ijspeert et al., 2007); (Sprewitz et al., 2008)),

$$\left. \begin{aligned} \dot{\varphi}_i &= 2\pi v_i + \sum_j r_j w_{ij} \sin(\varphi_j - \varphi_i - \phi_{ij}) \\ \ddot{r}_i &= a_i \left( \frac{a_i}{4} (R_i - r_i) - \dot{r}_i \right) \\ \theta_i &= r_i (1 + \cos(\varphi_i)) \end{aligned} \right\} (1)$$

where  $\theta_i$  is the output of oscillator  $i$ , which has amplitude  $r_i$ . Both the amplitude and the output are angles expressed either in radians or in degrees, which are subsequently sent to the motor controllers of the robot. By deriving the equation (1), we obtain three types of CPGs, Uncoupled, Unidirectional and Bidirectional CPGs respectively.

$$\left. \begin{aligned} \dot{\varphi}_1 &= 2\pi v_1 \\ \ddot{r}_1 &= a_1 \left( \frac{a_1}{4} (R_1 - r_1) - \dot{r}_1 \right) \\ \dot{\varphi}_2 &= 2\pi v_2 \\ \ddot{r}_2 &= a_2 \left( \frac{a_2}{4} (R_2 - r_2) - \dot{r}_2 \right) \end{aligned} \right\} (2)$$



then there are two fixed points; one of them is stable and the other one is unstable. The stability of the fixed point is determined by the sign of

$$\frac{df(\phi_\infty)}{d\phi} = -R_2 w_{12} \cos(\phi_\infty - \phi_{12}).$$

The fixed point is stable if this quantity is negative, and unstable if it is positive. If the initial phase difference is the unstable fixed point, the two oscillators will remain synchronized with that phase difference, hence there is no bifurcation.

The third case is Bidirectional Two CPGs. Let us consider four different cases:

*Case 1:*

Let us assume that  $\phi_{12} = -\phi_{21}$ ,  $w_{12} = -w_{21} = -w$ , and  $R_1 = R_2 = 1$ . Then, as  $t \rightarrow \infty$  we will have  $r_1 \rightarrow R_1$  and  $r_2 \rightarrow R_2$ .

$$\dot{\phi}_1 = 2\pi v_1 - w \sin(\phi_2 - \phi_1 - \phi_{12})$$

$$\dot{\phi}_2 = 2\pi v_2 + w \sin(\phi_1 - \phi_2 - \phi_{21})$$

Also, for  $\phi = \phi_2 - \phi_1$  which denotes the phase difference, the time evolution of the phase difference is determined by

$$\dot{\phi} = f(\phi) = \dot{\phi}_2 - \dot{\phi}_1 = 2\pi(v_2 - v_1).$$

Now, if  $f(\phi_\infty) = 0$ , then  $v_2 = v_1$ , which means there is no fixed point. In this case, it is said to *drift*.

*Case 2:*

Let us assume that  $\phi_{12} = -\phi_{21}$ ,  $w_{12} = w_{21} = w$  and  $R_1 = R_2 = 1$ . Then,

$$\dot{\phi} = f(\phi) = \dot{\phi}_2 - \dot{\phi}_1 = 2\pi(v_2 - v_1) - 2w \sin(\phi - \phi_{12}).$$

Now,  $f(\phi_\infty) = 0$  gives us

$$\phi_\infty = \arcsin\left(\frac{\pi(v_2 - v_1)}{w}\right) + \phi_{12}.$$

If the oscillators synchronize, they will do so at the fixed points  $\phi_\infty$ . Note that there is no fixed-point if

$$\left|\frac{\pi(v_2 - v_1)}{w}\right| > 1.$$

That is, when the difference of intrinsic frequencies is too large compared to the coupling weight  $w$ , the oscillators do not synchronize, and are said to *drift*. If, on the other hand,

$$\left|\frac{\pi(v_2 - v_1)}{w}\right| = 1,$$

then, there is a single fixed point  $\phi_\infty = \frac{\pi}{2} + \phi_{12}$  when  $v_2 > v_1$ , and  $\phi_\infty = -\frac{\pi}{2} + \phi_{12}$  when  $v_2 < v_1$ .

This solution is asymptotically stable, and the two oscillators will synchronize with that phase difference. Finally, if

$$\left|\frac{\pi(v_2 - v_1)}{w}\right| < 1$$

then, there are two fixed points; one of them is stable and the other one is unstable. The stability of the fixed point is determined by the sign of

$$\frac{df(\phi_\infty)}{d\phi} = -w \cos(\phi_\infty - \phi_{12}).$$

The fixed point is stable if this quantity is negative, and unstable if it is positive. If the initial phase difference is the unstable fixed point, then the two oscillators will remain synchronized with that phase difference.

*Case 3:*

Let us assume that  $\phi_{12} = -\phi_{21}$  and  $R_1 = R_2 = 1$ . Then

$$\dot{\phi} = \dot{\phi}_2 - \dot{\phi}_1 = 2\pi(v_2 - v_1) - (w_{21} + w_{12}) \sin(\phi - \phi_{12})$$

and  $f(\phi_\infty) = 0$  leads to the fixed point

$$\phi_\infty = \arcsin\left(\frac{2\pi(v_2 - v_1)}{w_{21} + w_{12}}\right) + \phi_{12}.$$

Note that there is no fixed-point if

$$\left|\frac{2\pi(v_2 - v_1)}{w_{21} + w_{12}}\right| > 1.$$

That is, when the difference of intrinsic frequencies is too large compared to the coupling weight  $w_{21} + w_{12}$ , the oscillators do not synchronize, and are said to *drift*. If

$$\left|\frac{2\pi(v_2 - v_1)}{w_{21} + w_{12}}\right| = 1$$

then, there is a single fixed point

$$\phi_\infty = \frac{\pi}{2} + \phi_{12} \text{ when } v_2 > v_1, \text{ and}$$

$$\phi_\infty = -\frac{\pi}{2} + \phi_{12} \text{ when } v_2 < v_1.$$

This solution is asymptotically stable, and the two oscillators will synchronize with that phase difference. Finally, if

$$\left| \frac{2\pi(v_2 - v_1)}{w_{21} + w_{12}} \right| < 1$$

then, there are two fixed points; one of them is stable and the other one is unstable. The stability of the fixed point is determined by the sign of

$$\frac{df(\phi_\infty)}{d\phi} = -(w_{21} + w_{12}) \cos(\phi_\infty - \phi_{12})$$

Again, the fixed point is stable if this quantity is negative, and unstable if it is positive. If the initial phase difference is the unstable fixed point, then the two oscillators will remain synchronized with that phase difference.

Case 4:

Let us take  $\phi_{12} = -\phi_{21}$ . In this case, we have

$$\dot{\phi} = \phi_2 - \phi_1 = 2\pi(v_2 - v_1) - (R_1 w_{21} + R_2 w_{12}) \sin(\phi - \phi_{12})$$

and  $f(\phi_\infty) = 0$  results in

$$\phi_\infty = \arcsin\left(\frac{2\pi(v_2 - v_1)}{R_1 w_{21} + R_2 w_{12}}\right) + \phi_{12}.$$

Note that there is no fixed-point if

$$\left| \frac{2\pi(v_2 - v_1)}{R_1 w_{21} + R_2 w_{12}} \right| > 1$$

That is, when the difference of intrinsic frequencies is too large compared to the coupling weight multiple by amplitude  $R_1 w_{21} + R_2 w_{12}$ , the oscillators do not synchronize and are said to *drift*. If

$$\left| \frac{2\pi(v_2 - v_1)}{R_1 w_{21} + R_2 w_{12}} \right| = 1$$

then, there is a single fixed point  $\phi_\infty = \frac{\pi}{2} + \phi_{12}$  when  $v_2 > v_1$ , and  $\phi_\infty = -\frac{\pi}{2} + \phi_{12}$  when  $v_2 < v_1$ . This solution is asymptotically stable, and the two oscillators will synchronize with that phase difference. Finally, if

$$\left| \frac{2\pi(v_2 - v_1)}{R_1 w_{21} + R_2 w_{12}} \right| < 1$$

There are two fixed points; one of them is stable and the other one is unstable. The stability of the fixed point is determined by the sign of

$$\frac{df(\phi_\infty)}{d\phi} = -(R_1 w_{21} + R_2 w_{12}) \cos(\phi_\infty - \phi_{12})$$

The fixed point is stable if this quantity is negative, and unstable if it is positive. As such, when the initial phase difference is the unstable fixed point, the two oscillators will remain synchronized with that phase difference.

### Optimizing Gait Generation

In this section, we will consider three cases, where each pattern generator outputs angular patterns for each joint. To evaluate gait generation, we need to find the optimal parameter sets by using central pattern generators, which explains how the angular of the hip and the knee should vary with time to generate motion along x-direction. For each case, parameter sets for the central pattern of each joint is given below:

$P_1 = \{a_1, v_1, R_1, a_2, v_2, R_2\}$ . Uncoupled Case

$P_2 = \{a_1, v_1, R_1, a_2, v_2, R_2, w_{12}, \phi_{12}\}$ . Unidirectional case

$P_3 = \{a_1, v_1, R_1, w_{12}, \phi_{12}, a_2, v_2, R_2, w_{21}, \phi_{21}\}$ . Bidirectional case.

(Nolfi & Floreano, 2000) and (Alexander, 1996) used genetic algorithms to find the optimal parameter sets. In this study, there is only one cost function utilized; the different walking patterns depend on this cost function.

$$J = -C_1 \sum_{k=1}^n x_b(k) + C_2 \left[ \sum_{k=1}^n (\theta_1^2(k) + \theta_2^2(k)) \right] / N, \quad (5)$$

$C_1, C_2 \in [0,1]$ , with  $C_1 + C_2 = 1$ ,  $n$  is the number of elements of position vector in simulation, and  $N$  is the length of the time. To maximize the displacement, or the velocity, we should minimize  $J$ . If  $C_2 = 0$ , then the aim is to maximize the displacement. However, if  $C_1 C_2 \neq 0$ , then there will be another cost function involving energy related terms in addition to the position. The goal is to minimize the energy while changing the position. Actually, this fact is available in biological locomotion (Alexander, 1996). The angular positions of the hip and knee joints are shaped during the optimization. These cost functions result in two different walking patterns. The first cost function presents walking pattern with large variations in joint, because only the displacement is emphasized in this function. However, the second one moves in +x direction with small angular variations of hip and knee joints.

Still, there are two constraints  $0 \leq \theta_1, \theta_2 \leq \pi$ . Figures 3 through 5 below show some gaits as a result of evolutionary optimization technique. Evolutionary optimization algorithms reveal the gait below in case constraints applied for joint angles. In this study, we used the hybrid function during the optimization. A hybrid function is an optimization function that runs after the genetic algorithm terminates in order to improve the value of the fitness function. The hybrid function uses the final point from the genetic algorithm as its initial point. You can specify a hybrid function in Hybrid function options. Specifically, we used Optimization Toolbox function at pattern search or fmincon, a constrained minimization function. The example first runs the genetic algorithm to find a point close to the optimal point and then uses that point as the initial point for pattern search or fmincon. Following gait optimization, we may conclude that locomotion is achievable by using the cost function  $J$  for the case of the uncoupled two CPGs such as in the Figure 3.a, 3.b and 3.c

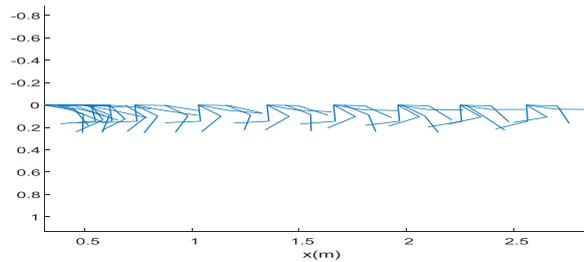


Figure 3a: Simulation of Walking Gait with Constraints

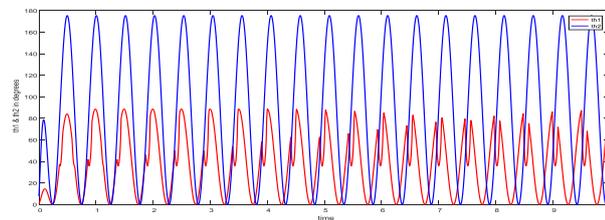


Figure 3b: Joint angles against Time

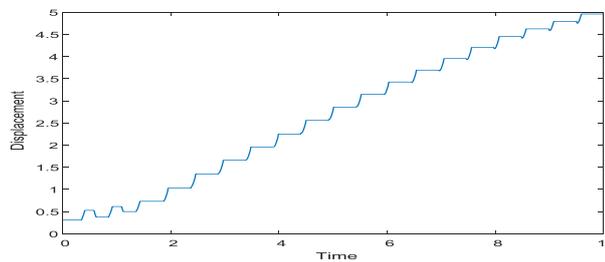


Figure 3c: Displacement against Time

Again, by utilizing gait optimization, stimulation of movement may be obtained using the cost function  $J$  for the case of the Unidirectional two CPGs, it is show by Figure 4.a, 4.b and 4.c

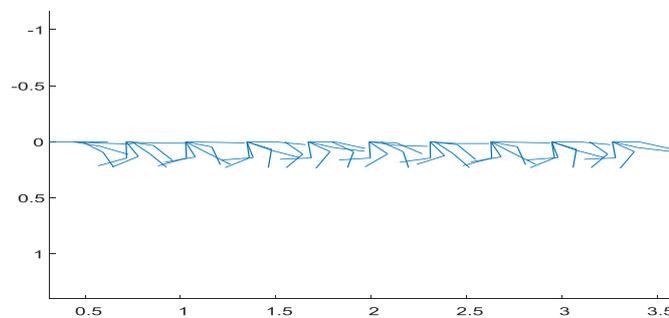


Figure 4a: Simulation of Walking Gait with Constraints

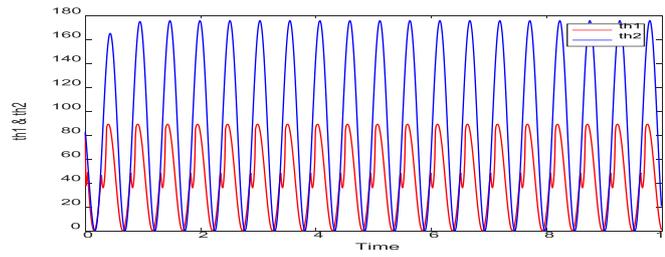


Figure 4b: Joint angles against Time

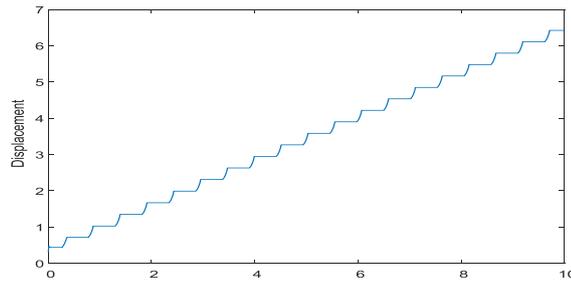


Figure 4c: Displacement against Time

Finally, by optimizing gait we obtain movement by means of using the cost function  $J$  for the case of the bidirectional two CPGs, figure 5.a, and 5.b show this results

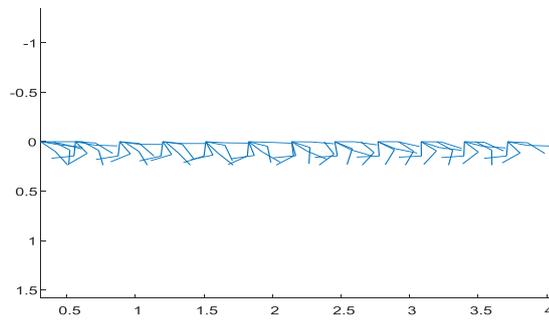


Figure 5a: Simulation of Walking Gait with Constraints

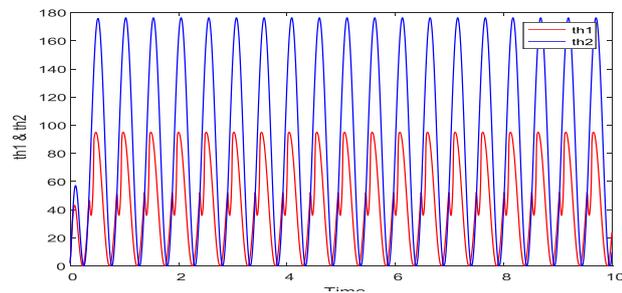


Figure 5b: Joint angles against Time

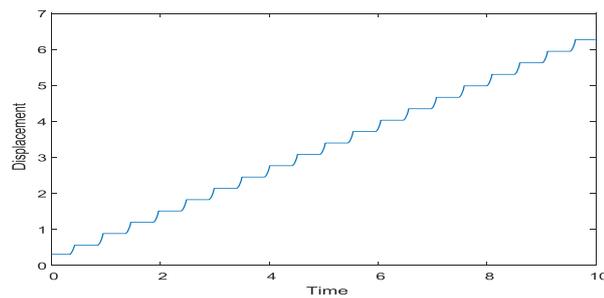


Figure 5c: Displacement against Time

Table 1 and 2 below summarize the results of the optimization in unbounded and bounded region. It is concluded that all parameters in three types of CPGs have positive values. The parameters  $R_1$  and  $R_2$  are the smallest values in both Tables. A close look at table 1 and 2, we clearly realize that in Table 1 the displacement and the velocity increase too much, hence it is not possible to be physically implemented, simply because optimization has been carried out in an unbounded region. By contrast, in Table 2, optimization can be physically implemented. Moreover, the three cases reveal no bifurcation; better results come from bidirectional two CPGs, though.

Table 1:

*Uncoupled, Unidirectional and Bidirectional Two CPGs in unbounded in 10 seconds*

Start at initial points				Uncoupled, Unidirectional and Bidirectional Two CPGs in unbounded in 10 seconds							
				Parameters values			Fval	Xb	Optimization type	E	
30.3617	28.5861	0.8646	20.3022	30.7363	28.5866	0.8628		-2.3849e+004	47.8770	GA & Hybridfcn at fmincon to uncoupled	3.0341
14.2796	1.2199			20.3314	14.2796	1.2202					
30.7363	28.5866	0.8628	20.3314	30.6689	28.5866	0.8629	20.3406	-1.1924e+004	47.8777	GA & Hybridfcn at fmincon to uncoupled	3.0343
14.2796	1.2202			14.2796	1.2202						
13.2211	27.9674	0.8541	23.3678	17.2865	40.4284	0.9287	28.0195	-2.8213e+004	112.8684	GA & Hybridfcn at patternsearch to unidirectional	2.9301
20.0000	1.2259	47.6153	50.7133	33.3458	1.2273	52.0449	50.3126				
59.3216	34.6978	0.8072	8.6457	61.4180	34.8193	0.8691	8.8225	-6.8743e+004	138.4982	GA & Hybridfcn at fmincon to Bidirectional	3.6014
1.1144	59.4932	34.2898	1.2267	1.0938	61.9136	34.4234	1.3595				
12.0076	6.7790			12.1180	6.7896						
61.4180	34.8193	0.8691	8.8225	62.4355	34.8369	0.8565	8.9123	-3.4536e+004	139.0181	Hybridfcn at fmincon unidirectional	3.6486
1.0938	61.9136	34.4234	1.3595	1.1130	61.9463	34.4603	1.3678				
12.1180	6.7896			12.1621	6.8044						
17.2865	40.4284	0.9287	28.0195	17.5270	40.6125	1.0038	27.1307	-5.7859e+004	116.9131	GA & Hybridfcn at patternsearch to unidirectional	3.0240
33.3458	1.2273	52.0449	50.3126	33.3437	1.2270	51.1525	50.1892				

Table 2:

*Optimizing Uncoupled, Unidirectional and Bidirectional Two CPGs in bounded region in 10 sec*

By optimizing of two CPGs		Optimizing Uncoupled, Unidirectional and Bidirectional Two CPGs in bounded region in 10 sec								
		Parameter's values					Fval	xb	Optimization type	E
D & E	Two Uncoupled without constraints	33.7958	1.9992	1.4355	68.5282	1.9857	-1.1020e+03	4.4082	GA	18.4589
		3.2592								
D & E	Two Uncoupled with constraints	18.6883	1.9928	0.7746	46.4124	1.9604	-1.3260e+03	4.9613	GA & Hybrid function at pattern search	4.1004
		1.5327								
D	Two Uncoupled with constraints	35.3887	1.9955	0.7564	26.4992	1.9606	-2.7312e+03	4.9856	GA & Hybrid function at fmincon	4.2406
		1.5707								
E	Two Uncoupled with constraints	0.0613	0.0426	0.0070	0.0309	0.0429	9.3082e-09	0.3100	GA & Hybrid function at pattern search	9.3175e-09
		0.0263								
D & E	Unidirectional Two CPGs	50.0000	1.9850	0.7804	13.1020	1.9230	-1.7074e+03	6.4220	GA & Hybrid function at pattern search	4.2509
		1.5356	2.0000	-0.3699						
D	Unidirectional Two CPGs	20.2328	1.9347	0.7369	25.6692	-1.9619	-3.0227e+03	5.5084	GA & Hybrid function at fmincon	4.1719
		1.5424	-0.3227	3.2986						
D & E	Bidirectional Two CPGs	48.2175	1.9592	0.8301	1.9690	-0.5784	-1.6230e+03	6.2665	GA & Hybrid function at pattern search	4.2799
		31.8414	1.9616	1.5398	1.7346					
D	Bidirectional Two CPGs	9.5946	1.9499	0.8009	1.2422	5.5546	-3.1376e+03	6.1787	GA & Hybrid function at fmincon	2.9796
		48.1560	1.9717	1.2744	1.9235					

Where E= Energy, D= Displacement, Fval= objective Function and xb=Displacement in meter

## Conclusion and Future Directions

To sum up, in this paper uncoupled, unidirectional and bidirectional two CPGs are used to generate motion for one leg with two degree of freedom. The study shows that when optimization is conducted in an unbounded region, the results are impossible to be implemented physically. Furthermore, by using genetic algorithms and hybrid functions, it seems that it is difficult to find a global region, because there is no bifurcation for the parameters in the three cases above. However, when we consider the stability analysis presented above with the objective of decreasing the variation between steps, it is vital that we control the amplitude and the frequency to obtain better results. Such results, we believe, can be implemented physically. Most important, the study reveals CPGs can control biped locomotion not only in animals but also in human beings. Future research should investigate whether CPGs can control other functions in human bodies, such as breathing, let alone the stimulation of the arm movement.

## References

- Alexander, R. M. (2003). *Principles of animal locomotion*. Princeton, NJ: Princeton University Press.
- Alexander, R. M. (1996). *Optima for animals*, Princeton, NJ: Princeton University Press.
- Amrollah, E., & Henaff, P. (2010). On the role of sensory feedbacks in Rowat-Selverston CPG to improve robot legged locomotion. *Frontiers in neurorobotics*, 4, 113.
- Aoi, S., & Tsuchiya, K. (2005). Locomotion control of a biped robot using nonlinear oscillators. *Autonomous robots*, 19(3), 219-232.
- Arikan, K. B., & Irfanoglu, B. (2011). A test bench to study bioinspired control for robot walking. *Journal of Control Engineering and Applied Informatics*, 13(2), 76-80.
- Billard, A., & Ijspeert, A. J. (2000). Biologically inspired neural controllers for motor control in a quadruped robot. In *Neural Networks, 2000. IJCNN 2000, Proceedings of the IEEE-INNS-ENNS International Joint Conference on* (Vol. 6, pp. 637-641). IEEE.
- Brown, T. G. (1911). The intrinsic factors in the act of progression in the mammal. *Proceedings of the Royal Society of London. Series B, containing papers of a biological character*, 84(572), 308-319.
- Bucher, D., Haspel, G., Golowasch, J., & Nadim, F. (2000). Central pattern generators. *eLS*.
- Büschges, A. (2005). Sensory control and organization of neural networks mediating coordination of multisegmental organs for locomotion. *Journal of neurophysiology*, 93(3), 1127-1135.
- Casasnovas, B., & Meyrand, P. (1995). Functional differentiation of adult neural circuits from a single embryonic network. *Journal of Neuroscience*, 15(8), 5703-5718.
- Endo, G., Nakanishi, J., Morimoto, J., & Cheng, G. (2005, April). Experimental studies of a neural oscillator for biped locomotion with QRIO. In *Robotics and Automation, 2005. ICRA 2005. Proceedings of the 2005 IEEE International Conference on* (pp. 596-602). IEEE.
- Ijspeert, A. J. (2008). Central pattern generators for locomotion control in animals and robots: a review. *Neural networks*, 21(4), 642-653.
- Ijspeert, A. J., & Crespi, A. (2007, April). Online trajectory generation in an amphibious snake robot using a lamprey-like central pattern generator model. In *Robotics and Automation, 2007 IEEE International Conference on* (pp. 262-268). IEEE.
- Ijspeert, A. J., Crespi, A., Ryczko, D., & Cabelguen, J. M. (2007). From swimming to walking with a salamander robot driven by a spinal cord model. *science*, 315(5817), 1416-1420.
- Qijun, Z., J. T. M. C., & Chengju, L. I. U. (n.d). Dynamic walking of AIBO with Hopf Oscillators. *Chinese Journal of Mechanical Engineering*, 24(4), 1.
- Siegelbaum, S. A., & Hudspeth, A. J. (2000). *Principles of neural science* (Vol. 4, pp. 1227-1246). E. R. Kandel, J. H. Schwartz, & T. M. Jessell (Eds.). New York, NY: McGraw-hill.
- Marbach, D. (2004). *Evolution and online optimization of central pattern generators for modular robot locomotion*. (Unpublished Master Thesis), Swiss Federal Institute of Technology Lausanne, Lausanne, Switzerland
- Matsuoka, K. (1987). Mechanisms of frequency and pattern control in the neural rhythm generators. *Biological cybernetics*, 56(5-6), 345-353.
- Nolfi, S., & Floreano, D. (2000). *Evolutionary robotics: The biology, intelligence, and technology of self-organizing machines*. Cambridge, US: The MIT press.
- Parker, G. A., & Smith, J. M. (1990). Optimality theory in evolutionary biology. *Nature*, 348(6296), 27-33.
- Pearson, K. G. (1995). Proprioceptive regulation of locomotion. *Current opinion in neurobiology*, 5(6), 786-791.
- Righetti, L., & Ijspeert, A. J. (2006, May). Programmable central pattern generators: An application to biped locomotion control. In *Robotics and Automation, 2006. ICRA 2006. Proceedings 2006 IEEE International Conference on* (pp. 1585-1590). IEEE.
- Sillar, K. T. (1996). The development of central pattern generators for vertebrate locomotion. *Advances in Psychology*, 115, 205-221.

- Sproewitz, A., Moeckel, R., Maye, J., & Ijspeert, A. J. (2008). Learning to move in modular robots using central pattern generators and online optimization. *The International Journal of Robotics Research*, 27(3-4), 423-443.
- Taga, G. (1998). A model of the neuro-musculo-skeletal system for anticipatory adjustment of human locomotion during obstacle avoidance. *Biological cybernetics*, 78(1), 9-17.
- Taga, G., Yamaguchi, Y., & Shimizu, H. (1991). Self-organized control of bipedal locomotion by neural oscillators in unpredictable environment. *Biological cybernetics*, 65(3), 147-159.
- Van Vreeswijk, C., Abbott, L. F., & Ermentrout, G. B. (1994). When inhibition not excitation synchronizes neural firing. *Journal of computational neuroscience*, 1(4), 313-321.